

## **Autonomous driving at intersections: combining theoretical analysis with practical considerations**

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### **Abstract**

The move towards automated driving is gaining impetus recently. This paper follows the approach of combining theoretical analysis with practical issues. It gives an insight of some practical problems that are encountered when running automated vehicles in real environments, using intersection crossing as a major example. The aim is not to try to be exhaustive but to show some criteria (safety, efficiency, reactivity, resilience, scalability...) for decision making in automated driving that have to be balanced before any mass deployment. In a second part we introduce mathematical tools that can help define algorithms and systems that improve current state of the art. We will also show some perspective for accommodating the hypotheses of these mathematical tools with real life constraints.

### **Keywords:**

Automated driving, Decision making, Intersection.

### **1. Introduction**



**Figure 1 Cybercars cooperating in Nancy (France).**

We focus on the decision making stage of the automated driving. We assume that perception and control

are reliable. The perception system gives us a clear picture of the environment, such as road geometry, blocking, other traffic participants, *etc.* The control system is also precise enough to realize the action that is decided.

Human is good at taking good driving decisions quickly in all environment during most of the time: remember that human caused accidents occur only once every million kilometres. Traffic rules help a lot in structuring the human decision making, *e.g.*, right or left priority, traffic signs, markings.... Moreover, there is also a shared way of driving: social rules, so that finally drivers cooperate most of the time. For now, automated vehicles are far behind human mind in decision making. Can we reproduce the decision making procedure of human drivers in automated vehicles? And a step further, can we enhance it?

To analyse this question we restrain the scope of this paper to intersections because this is the place that forces us to consider the diversity and complexity of real life. Intersection goes from rural crossings to urban intersections; flows may be various: pedestrians, cycles, cars, trucks, buses..., rules and social behaviour can be complex. Intersections concentrate about one third of all accidents in a very small road length. Nevertheless, we can still say that human driver in general handles well this challenge—sometimes by a very cautious move. To be widely accepted, automated driving will need to overcome this challenge that are not only about a single vehicle.

For all these reasons this paper focuses on the question of taking driving decisions for automated vehicles at intersection, when there are flows of automated vehicles, and in a second step when there are flows of other moving objects (*e.g.* human-driven cars or pedestrians). We believe that solving this problem will also help enhancing the decision making in other traffic scenarios.

The paper is organized as follows: Section 2 exposes several criteria for the system design of autonomous intersection that each needs to be raised to a minimum level but are contradictory. In Section 3 we review the existing literature on intersections for automated traffic and present our approach to autonomous intersection with some provable properties. We show that our approach balances several criteria. Since the world is not perfect, we consider the usability of such a framework in real life in Section 4. Finally we give a simple simulation and concludes the paper in Section 5 and 6.

## **2. Practical requirements and criteria for decision**

Decision making for an automated vehicle means to reason according to some general principles that is compliant with the use that is expected. Since the first expectation for passengers is to be safe, safety is an almost absolute criteria: one can find ethical cases where safety of one car is not prime but these are really extreme and can be left for the future. However, even with a good decision making we cannot avoid all crashes: others can cause accidents or there may be unexpected events. The decision making should not be over-cautious: a very slow speed could seem safer but is socially unacceptable since it slows the traffic. The efficiency of driving is also an expectation. We mean by efficiency both the ability to drive yourself quickly to your destination and to have a maximized global throughput of the flows of vehicles. Therefore safety and efficiency are both major criteria but cannot be optimized at the same

time. More precisely there is a balance to make between cautious and efficient driving. Note that this topic is highly related to perception: if you know perfectly the environment you can drive faster; unfortunately perception is never perfect so that there is a deep link between perception performance, safety and efficiency.

When considering more technically the decision making, another question is when do you have to take decision? This ranges from a highly planned long-term decision making to a short-term reactive scheme. When people drive, we take make long-term decisions (e.g. navigation) while also reacts to current traffic condition (braking, overtaking, *etc.*). The two processes, planning and reacting are both necessary but must be again balanced. Planning is necessary to compute trajectories that will be the input of the control part of an automated vehicle. But planning too much leads to rigid decision and re-planning will be necessary every time there is an unexpected event breaking the hypotheses of the planning. There are also drawbacks for a purely reactive policy: you will take into account only your close neighborhood and there is a risk of acting narrow-minded. Clearly we need to combine both approaches but it means you should react according to a plan to have a consistent decision making. Reactivity means also the ability to perform emergent manoeuvres to avoid the system to enter collision states, and then to start again moving if the problem is solved, that is, resilience.

Information on the ego vehicle and the environment is vital for viable decision making. Obtaining information requires the communication among different traffic participants. To avoid inconsistency of decisions and increase efficiency, a centralized decision making scheme is usually preferred that collects and processes information in a central entity. A good example is the traffic light: a centralized signal for coordinating flows. Nevertheless, getting the entire decision making process outside of the car is not practically possible because a communication failure would cause severe problem. Moreover, communication delays prevent us from reacting quickly enough to changing environment. This is why distributed systems have been considered for a long time. An additional advantage of distributed systems is the scalability: thanks to local interactions, they can expand to very large number of vehicles while a centralized architecture can have either communication or processing bottlenecks. A real system will have to balance between consistent decision making and efficiency that is linked to centralized systems, and autonomy and short reaction times that are linked to distributed systems.

A drawback of distributed systems is that most of the literature deals with homogeneous agents. However there are seldom two identical cars in a real street. Thus we have the problem of how homogeneous the system need to be. For communication devices there is a clear trend toward standards, but for decision making systems it is highly improbable that all decisions will be standardized. The question is to define to what extend manoeuvres should be standardized and where is the freedom.

Finally we have to decide the how vehicles interact with each other. It has been demonstrated that fully cooperative flows at intersection are highly efficient than no cooperative ones. However, such scheme does not necessarily mean that the interest of each individual vehicle is satisfied. The problem of fairness raises. We might finally be more attracted by egoistic decision making: keeping the autonomy of decision and preserving our own interest.

The main criteria we are considering in this paper are summarized in the table below.

**Table 1- Criteria and the related properties of the system**

Criteria that have to be balanced		Related properties of the system
Safety	Efficiency (traffic)	
Planning	Reactivity	Resilience
Centralized	Distributed	Scaling, autonomy, short reaction time
Homogeneous	Heterogeneous	Scaling, standardization
Cooperative	Self-optimized (egoistic)	Social behavior, fairness, autonomy

### 3. Intersections in automated traffic

As a major component of urban road network and a major bottleneck for traffic efficiency and safety, intersection attracts many researchers' attentions in last decades. The emergence of autonomous driving provides us an opportunity to reconsider how intersections could be updated to profit from the automation of road traffic. As the cost of modifying geometry of the intersection, or constructing new intersections is prohibitively high, we mainly focus on approaches that re-utilize the current infrastructure. To accommodate the technological advancements in Intelligent Transportation Systems (ITS), we allow some minor updates on intersection infrastructure (e.g. sensors, road-side unit). In Section 3.1, we review the current literature with the criteria identified in Section 2. We show that most existing work cannot strike a good balance between criteria. In Section 3.2, we describe our approach based on the priority-based framework [1] that overcomes some short-backs of the existing literature.

#### 3.1 Literature review

Traffic lights are installed in many intersections to coordinate conflicting traffic flows and ensure the road safety. However, the efficiency and safety of such system is doubted: 44% of collisions in the U.S. are within the intersection area (equipped or not) and delays induced by traffic lights can be high. A hot research topic is to design flexible and intelligent traffic light systems that can adapt the signal timing to various context: the magnitude of incoming flows [2], queue length [3], [4], the state of traffic lights upstream or downstream [5], etc. Traffic lights strike a good balance between reactivity *vs.* planning, centralized *vs.* distributed and homogeneous *vs.* heterogeneous. Traffic lights define high level priority rules (vehicles can proceed if in a green phase) and vehicles can reactively cross the intersection, retaining a large autonomy. The phase switching design is performed centrally in the infrastructure while the execution is performed by every individual vehicle. Heterogeneous flows can be considered by properly tuning the parameters of light. However, although optimized, the efficiency of traffic lights is still improvable in low and medium traffic load (in high or near-saturated load, traffic lights are in fact

efficient as the incoming vehicles can be considered as continuous flows and the control with less phase switching is efficient).

In [6], the concept of autonomous intersection is proposed. Automated vehicles are coordinated to cross the intersection without traffic light, fully utilizing the advanced sensing, communication and manoeuvre capacities of vehicles. A major advantage of such approach is its efficiency improvement (throughput, average delay, etc.) compared with traffic lights. Consequently, the concept is widely accepted by researchers and many approaches are proposed [7]–[14]. We briefly discuss them with the criteria of Section 2 in mind. The existing approaches can be categorized into planning-based approaches [6], [8], [9] and hybrid approaches [10]–[14].

In planning-based approaches, an intersection controller finds in a first phase collision-free trajectories for all vehicles (usually in centralized way); then in a second phase vehicles should follow the trajectories to cross the intersection. The second phase can be considered as pseudo-decentralized as the control inputs are required to be exactly implemented in vehicles. Any deviation from programmed trajectories may cause collisions. For example, reference [9] formulates the trajectory generation problem as a large-scale nonlinear constrained optimization problem and uses optimization tools to solve the problem. In [6], vehicles can reserve trajectories from the intersection controller and the controller will centrally decide to accept or deny the reservations. These centralized, planning-based approaches are quite efficient, while the system properties like resilience, scaling, autonomy and short reaction time are not ensured. Heterogeneity is also difficult to integrate in planning-based approaches as this will necessarily introducing a large number of parameters for different vehicles, adding more complexity to already complex system. Finally, vehicles are assumed to be fully cooperative to achieve the goals of the intersection controller. The egoistic need of each individual vehicle might be ignored, although some optimizations in the intersection controller might ameliorate the fairness property of the system.

Hybrid approaches only plan some high level priority relations between vehicles in centralized or decentralized way. Such priority relations usually define the relative orders of vehicles to cross the collision points. Vehicles are usually able to cross the intersection in reactive ways while obeying these relations. For example, [10] proposes an approach based on navigation function. Potentials are defined over the workspace (intersection). The gradient of the navigation function towards the destination and high potential are used to mark the vehicles that the ego-vehicle must defer to. These approaches strike balance between planning and reactive, centralized and distributed. If priority relations are properly defined, the balance between cooperation and egoistic need can also be achieved: high-level relations are used to optimize the objective of intersections while egoistic objectives can be achieved by each vehicle with the premise of respecting the high-level relations. It is clear that the efficiency is sacrificed to some extent compared with planning based approaches in trade of autonomy. Finally, hybrid approaches that properly balance planning and reactive behaviours do not automatically ensure the resilience of the system. Some solutions try to avoid collisions by forcing a minimal time distance [10] or by penalizing the case that two vehicles are too close [14]. These methods may avoid collision under normal situation, while the resilience guarantee is fragile: there is no insurance that vehicles won't collide if a vehicle unexpectedly brakes due to the presence of a pedestrian. Some other approaches

[11]–[13] adopt a more control theoretical approach to formulate the safety constraints. The technique of reachability analysis is usually used. A bad set that comprises all collision configurations can be defined. Vehicles are then controlled in a way that they will always avoid the bad set. In other words, the reachable set of current system state (parallel composition of configurations of all vehicles) should not intersect with the bad set. The advantage of such approaches is that the resilience of the system is theoretically provable, under some mild technical assumptions.

To conclude the analysis above, autonomous intersections using hybrid approach with provable resilient property seem to be an acceptable solution for intersections in automated traffic. In the following, we present our approach to autonomous intersections that properly balances among different criteria.

### 3.2 A priority-based approach to autonomous intersection

We adopt the priority-based framework [1] to tackle the autonomous intersection problem. Considering a collection of automated vehicles  $N$  at an intersection. Define  $L$  the set of possible paths to cross the intersection. Every vehicle  $i \in N$  is constrained to forward motion on a fixed path  $l_i \in L$ . Define  $x, v, u$  respectively as the position and velocity of a vehicle. Define  $s = (x, v)$  as the state of the vehicle. The dynamics of a vehicle is supposed to be discrete-time double integrator, ignoring the lateral control of vehicles

$$s^+ = f(s, u), x \in X, v \in V := [0, \bar{v}], u \in U := [\underline{u}, \bar{u}]$$

where  $X, V, U$  are respectively the set of possible positions, velocities and control inputs. Define  $\phi(t, s, \mathbf{u})$  as the flow of the system, where  $\mathbf{u} \in S(U)$  is the control signal overtime. Define  $\phi_x$  as the projection of system flow on the axis of the position.

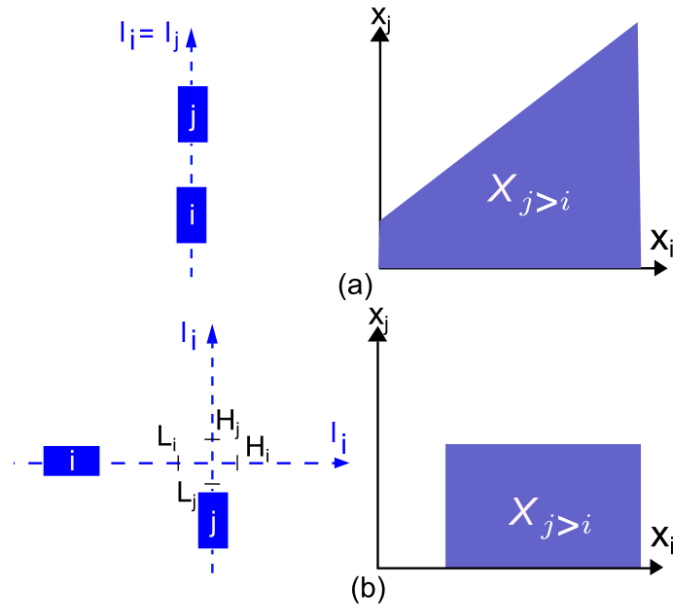
We define the priority relation  $>$  that denotes the relative order of two vehicles. Given two vehicles  $i$  and  $j$ ,  $j > i$  means that vehicle  $i$  should defer to vehicle  $j$  if any conflict. Define  $X_{j>i}$  the set of inadmissible configurations that violate the priority relation (note that the set of collisions is a subset of  $X_{j>i}$ ). There are mainly two possible cases for  $X_{j>i}$ :

- (Figure 2a)  $j$  is the direct predecessor of  $i$ :

$$X_{j>i} = \{(x_i, x_j) \mid x_i - x_j > L\}$$

- (Figure 2b)  $j$  is in different path of  $i$  but is in conflict:

$$X_{j>i} = \{(x_i, x_j) \mid x_i > L_i \text{ and } x_j < H_j\}$$



**Figure 2 Completed obstacle region of two different cases**

The coordination problem is then separated into a high-level priority assignment problem and a vehicle control problem under fixed priorities. Priority assignment decides the relative priorities of vehicles to cross the intersection. Vehicles are then controlled to respect priorities, that is, the configuration of any couple of conflicted vehicles remain out of  $X_{j>i}$ . If we consider vehicles as vertices and priority relations as arcs, a priority graph can be constructed. The introduction of the priority relations as a theoretical tool brings several benefits [1]:

- (Deadlock-free). If there is no loop of priorities, then the intersection system is deadlock-free.
- (Autonomy). A priority graph uniquely defines a homotopy class of trajectories in the vector space of  $X_{free} = \{(x_i)_{i \in N} \mid \forall i, \forall j > i, (x_i, x_j) \notin X_{j>i}\}$ . Thus vehicles are guaranteed to have some autonomy since the homotopy class is not empty.
- (Efficiency guarantee). If we assume that vehicle arrival times and vehicle crossing times can be estimated (even roughly with upper bound), the priority assignment problem is equivalent to a polling system, where a single server serves different queues. In [15], the author has demonstrated that the delay of the intersection can be no more than the corresponding polling system. Moreover, standard optimization techniques for polling system can be used to optimize the priority assignment process.

We now consider the problem how vehicles can be controlled to respect priorities. We note that simply demanding a lower priority vehicle to cross the collision point later than its prior vehicles does not guarantee the system property of resilience: any unexpected brake of a prior vehicle may cause collisions as the lower priority vehicle may not be able to stop immediately due to momentum. We seek a stronger priority preserving property defined as follows.

$$\forall j > i, \exists \mathbf{u}_i \in S(U_i), \text{ s.t. } \forall t > 0, \forall \mathbf{u}_j \in S(U_j), (\phi_{x,i}(t, s_i, \mathbf{u}_i), \phi_{x,j}(t, s_j, \mathbf{u}_j)) \notin X_{j>i}$$

That is, for vehicle  $i$ , it should be controlled in a way that priorities are respected regardless the control of its prior vehicles.

We have demonstrated [16] that this priority preserving property can be achieved if the following condition is achieved:

$$\forall j > i, f(s_i, u_i) \in B_{j>i}(s_j)$$

where

$$B_{j>i}(s_j) = \{s_i \in S_i | \forall t > 0, (\phi_{x,i}(t, s_i, \underline{\mathbf{u}}_i), \phi_{x,j}(t, s_j, \underline{\mathbf{u}}_j)) \notin X_{j>i}\}$$

$\mathbf{u}$  is maximal brake command over time.

With the property formulated above, given the state of prior vehicles and the current state of ego vehicle  $s_i$ , the upper bound of admissible control input  $u_i^{adm}$  can be calculated. Thus if every vehicle are controlled with a control value in  $[\underline{\mathbf{u}}, u^{adm}]$  at every time instant, the system is not only safe, but also resilient to unexpected brakes. Finally, the control of each vehicle can be formulated as an optimal control problem under priority preserving constraints. A MPC-based approach [16] can be adopted to solve the problem.

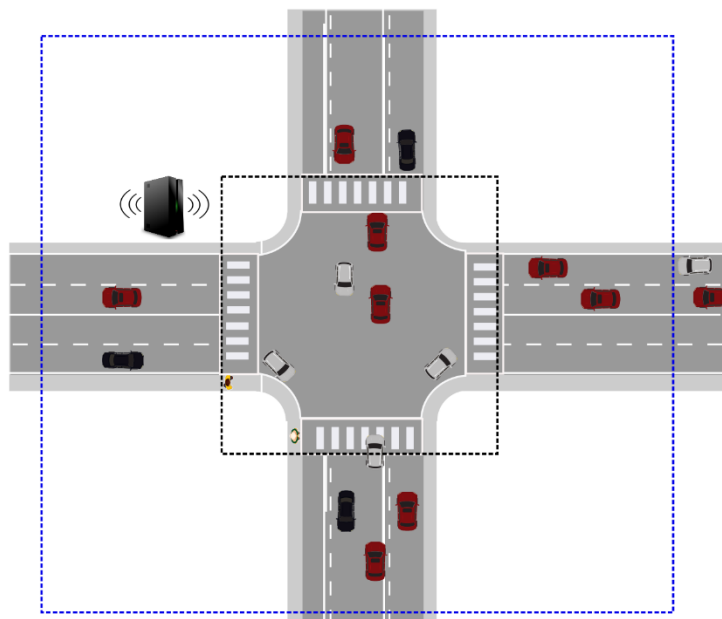
To conclude, the priority-based approach strike good balance between efficiency and safety, as the system resilience is guaranteed and the delay of each individual vehicle is less than the corresponding polling system. The balance between planning and reactive, centralized and distributed are also achieved through the hybrid structure. Cooperative behaviour is achieved through the respect of priority preserving conditions and personalized interests can be ensured through optimizing the control under priority preserving constraints.

### 3.3 Autonomous intersection design

Now we have introduced the mathematical foundation of our approach. We discuss the implementation aspect in this section.

Consider the intersection in Figure 3. A roadside unit serves as the intersection controller. It is responsible for affecting priorities to incoming vehicles. The area within black dashed box is the intersection area. Vehicles are not allowed to enter the intersection area if they are not admitted (they are not assigned with priorities). The blue dashed box (the outer one) refers to the cooperative area, where vehicles start to communicate and cooperate with the intersection controller.





**Figure 3 The design of an autonomous intersection.**

The system works as follows:

- Vehicles that enter the cooperative area notify the intersection controller with their presence.
- The intersection controller maintains a priority graph (for admitted vehicles) and a waiting list for non-admitted vehicles. The controller works in discrete time. At each time step, one or several vehicles will be admitted and assigned with priorities according to a priority assignment policy. Such policy can be adapted from the polling system.
- Admitted vehicles are allowed to autonomously cross the intersection under the assigned policy. Non-admitted vehicles are required to avoid entering the intersection area.
- Once vehicles leave the intersection, they are removed from priority graph and the corresponding priority relations are revoked.

#### **4. From theory to practice**

The above presented framework requires strong assumption: perception is perfect, communication is without delay, control is simple and there is only automated vehicles with the same dynamics and obeying the rules. The aim of this section is to show that how the algorithms derived from this framework can still work in a less constraint environment.

There are many possibilities of failures in both perception and control. One can consider a perception failure as an unexpected event when the information arrives too late with respect to a normal functioning. In our framework, since all driving states are brake-safe, there will be no collision between the automated cars if a car brakes when it has perception failures. A more severe issue is non-detection of objects. The only solution to this problem is to avoid non-detection in critical places: monitoring critical

areas (where the vehicle will be in the next seconds) is indeed a research topic in perception. The control failure is worse since if the vehicle fails to respond properly to commands, there is not much to do. The current approach is not to fail silently but to have degraded control modes for which either the vehicle can still exit the intersection, or can get a higher priority so that other vehicles let this failing vehicle leave the intersection (before stopping safely). Again this can be handled as a kind of unexpected event leading to brake to keep brake-safe property.

For the communication delays, our framework is more straightforward since time is slotted: if communication delays are less than slot duration, there may be slightly outdated information but the objectives of car-to-car communication delays (car-to-traffic lights would be the same) are in fact far less (10-20 ms) than control time slots (typically 50-200 ms). A shutdown of the communication network would be more damaging since no allocation would be given but would not directly lead to collisions. Another challenge is the heterogeneity in low level vehicle dynamics: will brake-safe states still be well defined for certain dynamics? A possible solution is to forbid automated vehicles that would not pass some tests in order to have bounds on critical control properties. Heterogeneity is also about various flows: human driven cars, pedestrian, motorbikes... For this we can still use the “unexpected event” strategy but it is viable only for tiny flows of non-controlled traffic participants. If you want to go beyond that point, we need to know somehow what are their goals and probable trajectories of these objects.

Finally, we have to consider more generally how a traffic participant will react to other participants. Unfortunately this reaction is not always driven by deterministic rules but can be complex strategy, such as game theory. The main factor in getting game theoretical concepts into planning is how cooperative can we expect other traffic participants. Statistically drivers and pedestrian are very collaborative. But there are some egoistic behaviours and it is very difficult to know in advance how far we can assume the cooperative willingness of a detected “object”. Fully non-cooperative is mathematically intractable. The study on the interactions of traffic participants is very active with various, sometimes very original, research topics.

## 5. Simulation

In this section, we provide a demonstration of our proof-of-concept simulator. The intersection is a 4-leg intersection in a 500x500 area. Each incoming single-lane road is 250 meter long. The cooperative area is 100x100 and the intersection area is 10x10. The simulator is running with a time step of 0.5 second. We adopt a priority assignment policy described in [1] and the vehicle is controlled using a MPC controller described in [16]. Basic vehicle dynamics are listed in the following table. A simulation video with the inflow 1200 veh/h is available in this [link](#). We observe that vehicles can cross the intersection almost without delay. We remind that this simulator is of its simplest form. We are planned to implement our approach in a widely accepted traffic simulator VISSIM and the preliminary implementation has already started.

**Table - Vehicle dynamics**

Vehicle size	5m length, 3m width
Maximal acceleration	3.0 m/s <sup>2</sup>
Maximal deceleration	-4.5 m/s <sup>2</sup>
Maximal speed	10 m/s
Desire speed	10 m/s

## 6. Conclusion

This paper discuss the decision making problem of automated vehicles, using autonomous intersection as an example. We exposed the problems that we encountered in previous research and in demonstrations (European Cybercars, Cybercars-2, CityMobil, CytiMobil-2, CityNetMobil, French project MobiVIP...). Our experience of real vehicles in urban environment guided our considerations. Then we propose a theoretical framework, combining priority allocation for planning and brake-safe control for reacting. In this framework we are able to prove desirable properties like safety (no collision, robustness to sudden decelerations), efficiency (no deadlock) and resilience (the system is able to restart in case of a full system stop). We have shown that it is possible to relax some assumptions and keep these properties. After this theoretical insight, we went back to the real world and we have shown that the framework can drive our thinking of future autonomous intersections. However there remains hard problems, such as dealing with unpredictable, maybe non-cooperative “objects” and the concept of fairness.

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